# Worksheet 3

Sections 306 and 310
MATH 54

August 30, 2018

**Exercise 1.** Do the following vectors span $\mathbb{R}^3$?

$$\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$$

**Exercise 2.** Do the following vectors span $\mathbb{R}^4$?

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

**Exercise 3.** Determine if the following systems have a nontrivial solution:

- $x_1 - 3x_2 + 7x_3 = 0$, $\qquad -2x_1 + x_2 - 4x_3 = 0 \qquad x_1 + 2x_2 + 9x_3 = 0$
- $-5x_1 + 7x_2 + 9x_3 = 0$, $\qquad x_1 - 2x_2 + 6x_3 = 0$

**Exercise 4.** Describe all solutions of $A\mathbf{x} = \mathbf{0}$, for the following matrices. Express your answers in parametric vector form.

$$A = \begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

**Exercise 5.** Describe the solutions of the system given by the following augmented matrix. Express your answer in parametric vector form.

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{bmatrix}$$

**Exercise 6.** Let $A$ be a $3 \times 2$ matrix with 2 pivot positions. Does $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution? Does $A\mathbf{x} = \mathbf{b}$ have at least 1 solution for every $\mathbf{b}$ in $\mathbb{R}^2$?