# Worksheet 26

Sections 306 and 310
MATH 54

Nov 27, 2018

**Exercise 1.** Find a general solution of the system $\mathbf{x}'(t) = A\mathbf{x}(t)$ for the given matrix A. You can use the fact that the eigenvalues of $A$ are 2, $2+i$, and $2-i$.

$$A = \begin{bmatrix} 5 & -5 & -5 \\ -1 & 4 & 2 \\ 3 & -5 & -3 \end{bmatrix}$$

**Exercise 2.** For the coupled mass-spring system governed by (10) on page 536 of the book, assume $m_1 = m_2 = 1\text{kg}$, $k_1 = k_2 = k_3 = 1$ N/m and assume initially that $x_1(0) = 0$ m, $x_1'(0) = 0\text{m/s}$, $x_2(0) = 2\text{m}$ and $x_2'(0) = 0$ m/s. Find the normal frequencies and solve the intitial value problem.

**Exercise 3.** Determine whether the given vector functions are linearly independent of linearly dependent on $(-\infty, \infty)$.

(a) Show that the Cauchy-Euler equation $at^2y'' + bty' + cy = 0$ can be written as system

$$t\mathbf{x}' = A\mathbf{x}$$

with a constant coefficient matrix $A$, by setting $x_1 = y/t$ and $x_2 = y'$.

(b) Show that for $t > 0$ any system of the form above has nontrivial solutions of the form $\mathbf{x}(t) = t^r\mathbf{u}$ if and only if $r$ is an eigenvalue of $A$ and u is a corresponding eigenvector.