

Worksheet 25

Sections 306 and 310
MATH 54

Nov 25, 2018

Exercise 1. Consider $\mathbf{x}'(t) = A\mathbf{x}(t)$, $t \geq 0$, where $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$.

- (a) Show that the eigenvalues are -1,-3 and $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are the corresponding eigenvectors.
- (b) Sketch the trajectory of the solution having initial vector $\mathbf{x}(0) = \mathbf{u}_1$.
- (c) Sketch the trajectory of the solution having initial vector $\mathbf{x}(0) = -\mathbf{u}_2$.
- (d) Sketch the trajectory of the solution having initial vector $\mathbf{x}(0) = \mathbf{u}_1 - \mathbf{u}_2$.

Exercise 2. Give a general solution to:

$$\mathbf{x}'(t) = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$$

Exercise 3. Use the substitutions $x_1 = y$, $x_2 = y'$ to convert $ay'' + by' + cy = 0$ into a normal system. Show that the characteristic equation of the system is the same as the aux equation for the original equation.