Worksheet 24

Sections 306 and 310
MATH 54

Nov 13, 2018

Exercise 1. Write the given system in normal matrix form: $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$.

$$x'(t) = x + y + z$$

$$y'(t) = 2x - y + 3z$$

$$z'(t) = x + 5z + e^{5t}$$

Exercise 2. Rewrite the given equation as a first order system in normal form:

$$y''' - y' + y = \cos(t)$$

Exercise 3. Determine whether the given vector functions are linearly independent of linearly dependent on $(-\infty, \infty)$.

(a) $e^t \begin{bmatrix} 1 \\ 5 \end{bmatrix}, e^t \begin{bmatrix} -3 \\ -15 \end{bmatrix}$

(b) $\begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}, \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$

Exercise 4. The given vector functions are solutions to a system $\mathbf{x}'(t) = A\mathbf{x}(t)$. Do they form a fundamental set? If so, find a fundamental matrix and five a general solution.

$$\mathbf{x_1}(t) = e^{-t} \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \qquad \mathbf{x_2}(t) = e^{4t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Exercise 5. Let $X(t)$ be the fundamental matrix for the system $\mathbf{x}'(t) = A\mathbf{x}(t)$. Show that $\mathbf{x}(t) = X(t)X^{-1}(t_0)\mathbf{x_0}$ is the solution to the initial value problem $\mathbf{x}' = A\mathbf{x}$, and $\mathbf{x}(t_0) = \mathbf{x_0}$.