Worksheet 21

Sections 306 and 310
MATH 54

Nov 1, 2018

Exercise 1. Find a general solution to the given differential equations:

(a) $y'' + y = 0$

(b) $y'' - 10y' + 26 = 0$

(c) $y'' - 4y' + 7y = 0$

Exercise 2. To see the effect of changing the parameter b in the initial value problem

$y'' + by' + 4y = 0$; $y(0) = 1$; $y'(0) = 0$

Solve the problem for $b = 5$, 4, and 2 and sketch the solutions.

Exercise 3. Find a general solution to the following higher-order equation:

$$y''' - y'' + y' + 3y = 0$$

Exercise 4. Prove the sum of angles formula for the sine function by following these steps. Let x be a fixed constant.

(a) Let $f(t) = \sin(x + t)$. Show that $f''(t) + f(t) = 0$, $f(0) = \sin x$, and $f'(0) = \cos(x)$.

(b) Use the auxiliary techniqe to solve the initial value problem $y'' + y = 0$, $y(0) = \sin(x)$, and $y'(0) = \cos(x)$.

(c) By uniqueness, the solution in part (b) is the same as $f(t)$ from part (a). Write this equality, this should be the standard sum of angles formula for sin(x+t).