Worksheet 20

Sections 306 and 310
MATH 54

October 30, 2018

Happy Halloween!!!

Exercise 1. Find a general solution to the given differential equations:

(a) $y'' - y' - 2y = 0$

(b) $y'' - 5y' + 6y = 0$

(c) $4y'' - 4y + y = 0$

Exercise 2. Solve the initial given value problems:

(a) $y'' + y' = 0$; $y(0) = 2$; $y'(0) = 1$

(b) $y'' - 4y' + 4y$; $y(1) = 1$; $y'(1) = 1$

Exercise 3. First-Order Constant-Coefficient Equations

(a) Substituting $y = e^{rt}$, find the auxiliary equations for the first-order linear equation $ay' + by = 0$, where a and b are constants with $a \neq 0$.

(b) Use the result from part 9a) to find the general equation of this first-order equations.

(c) What is the general solution to $3y' - 7y = 0$?

Exercise 4.   (a) With your group, reread definition 1 in this section (restated below):

A pair of functions $y_1(t)$ and $y_2(t)$ is said to be linearly independent on an interval I if and only if neither of them is a constant multiple of the other on all of I.

(b) Are $y_1(t) = e^{3t}$ and $y_2(t) = e^{-4t}$ linearly independent?

(c) Are $y_1 = \tan^2(t) - \sec^2(t)$ and $y_2(t) = 3$ linearly independent?

Exercise 5. (a) Explain why two functions are linearly dependent on an interval I if and only if there exist constants c_1 and c_2, not both zero, such that

$$c_1 y_1(t) + c_2 y_2(t) = 0$$

for all t in I.

(b) Discuss with your group how this connects to the idea of linear independence that we discussed in the linear algebra section of the course.

(c) Expand the defintion of linear independent functions to apply to sets of 3 or more functions.

Exercise 6. We are starting a new section of this course, so there may be vocabulary that you have not seen before. In the context of this chapter, make sure you know what each of the words/phrases means in your own words:

(a) an interval I

(b) general solution

(c) initial value problem

(d) uniqueness