Worksheet 2

Sections 306 and 310
MATH 54

August 28, 2018

Exercise 1. Find the general solution of the systems whose augmented matrices are shown below. Some of these matrices may look familiar :).

$$\begin{bmatrix} 1 & 4 & 0 & 1 \\ 2 & 7 & 0 & 10 \end{bmatrix} \qquad \begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$$

Exercise 2. Find h, k such that the system below has: (a) no solutions, (b) a unique solution, and (c) infinitely many solutions.

$$x + hy = 2$$

$$4x + 8y = k$$

Exercise 3. Solve the following systems:

- $x_1 - 3x_2 + 4x_3 = -4, \quad 3x_1 - 7x_2 + 7x_3 = -8, \quad -4x_1 + 6x_2 - x_3 = 7$
- $x_1 - 3x_2 = 5, \quad -x_1 + x_2 + 5x_3 = 2, \quad x_2 + x_3 = 0$

Exercise 4. A system of linear equations with more equations than unknowns is sometimes called *overdetermined*. Can such a system be consistent? Illustrate your answer with a specific system of 3 equations and 2 unknowns. (It may be helpful to draw a picture in the plane!)

Exercise 5. Write the following vector equation as a matrix equation and also as a system of linear equations. Solve the system.

$$x_1 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Exercise 6. Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}$.

$$\mathbf{a_1} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \mathbf{a_2} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \mathbf{a_3} = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ 8 \end{bmatrix}$$

Exercise 7. Write the following products as linear combinations of the columns of the matrix. Use this to compute the product.

$$\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Exercise 8. Write the augmented matrix for the linear system that corresponds to the matrix equation $Ax = \mathbf{b}$. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$