Worksheet 18

Sections 306 and 310
MATH 54

October 23, 2018

Exercise 1. Consider $\mathbb{P}_2$ with the following inner product:

$$\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$$

Let $p(t) = 3t - t^2$ and $q(t) = 3 + 2t^2$. Compute the following:

(a) $\langle p, q \rangle$

(b) $||p||$, $||q||$

(c) $\text{proj}_W q$, where $W = \text{span}\{p\}$.

Exercise 2. Use the axioms of inner product spaces to prove the following: For any vector $\mathbf{v}$ in an inner product space V,

$$\langle \mathbf{v}, \mathbf{0} \rangle = \langle \mathbf{0}, \mathbf{v} \rangle = 0$$

Exercise 3. True of false! Justify!

(a) There are symmetric matrices that are not orthogonally diagonalizable.

(b) An orthogonal matrix is always orthogonally diagonalizeable.

(c) The dimenaion of the eigenspace is sometimes less than the multiplicity of the corresponding eigenvalue.

(d) The dimension of an eigenspace of a symmetrix matrix is sometimes less than the multiplicity of the corresponding eigenvalue.

Exercise 4. The following matrix has eigenvalues $\lambda = -2, 7$. Orthongally diagonalize the matrix:

$$A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

Exercise 5. Use the Cauchy-Schwarz inequality to show that $\left(\frac{a+b}{2}\right)^2 \leq \frac{a^2+b^2}{2}$. HINT: Use the vectors $\begin{bmatrix} a \\ b \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$