

Worksheet 17

Sections 306 and 310
MATH 54

October 18, 2018

Exercise 1. Find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ by using the normal equations for $\hat{\mathbf{x}}$

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$$

Exercise 2. True and false! Justify your answers! A is $m \times n$ and \mathbf{b} is in \mathbb{R}^m .

- (a) If \mathbf{b} is in the column space of A , then every solution of $A\mathbf{x} = \mathbf{b}$ is a least-squares solution.
- (b) The least-squares solution of $A\mathbf{x} = \mathbf{b}$ is the point in the column space of A closest to \mathbf{b}
- (c) A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is a list of weights that when, applied to the columns of A , produces the orthogonal projection of \mathbf{b} onto $\text{Col } A$.

Exercise 3. Find the orthogonal projection of \mathbf{b} onto $\text{Col } A$ and use this to find a least-squares solution of $A\mathbf{x} = \mathbf{b}$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$$

Explain why this method would be more difficult for the matrix given in exercise 1.