# Worksheet 16

Sections 306 and 310
MATH 54

October 16, 2018

**Exercise 1.** Write $\mathbf{y}$ as the sum of two orthogonal vectors, one in span$\{\mathbf{u}\}$ and one orthogonal to it.

$$\mathbf{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Compute the distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and the origin.

**Exercise 2.** True and false! Justify your answers!

(a) If $A$ is an $n \times n$ matrix with orthogonal columns, then it is invertible.

(b) If a set $\{\mathbf{u_1}, \ldots \mathbf{u_p}\}$ has the property that $\mathbf{u_i} \cdot \mathbf{u_j} = 0$ whenever $i \neq j$ then $S$ is an orthnormal set.

(c) If $c$ is not 0, then the orthogaonl projection of $\mathbf{y}$ onto a vector $\mathbf{u}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{u}$.

**Exercise 3.** Let $W$ be the subspace spanned by the $\mathbf{v}'s$ and write $\mathbf{y}$ as a sum of a vector in $W$ and a vector orthogonal to $W$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

What is the closest point in $W$ to $\mathbf{y}$?

**Exercise 4.** Find an orthogonal basis for $\text{col}(A)$.

$$A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

**Exercise 5.** Without looking at the proof in the book, show that a set of nonzero orthogonal vectors is linearly independent.