Worksheet 15

Sections 306 and 310
MATH 54

October 11, 2018

Exercise 1. Find a unit vector in the direction of the given vector. Draw a picture of what an orthogonal vector would look like.

$$\begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$$

Exercise 3. True and false! Justify your answers!

(a) For any scalar c, $||c\mathbf{v}|| = c||\mathbf{v}||$.

(b) If $\mathbf{v}$ is orthoganal to every vector in a subspace W, then $\mathbf{v}$ is in $W^{\perp}$.

(c) If $||\mathbf{u}||^2 + ||\mathbf{v}||^2 = ||\mathbf{u} + \mathbf{v}||^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

(d) For an $m \times n$ matrix A, vectors in nul A are orthogonal to vectors in row A.

Exercise 3. Show that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ form an orthoganal basis for $\mathbb{R}^3$. Then express $\mathbf{x}$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

Exercise 4. For what values of b is the following matrix diagonalizable?

$$\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$