Worksheet 14

Sections 306 and 310
MATH 54

October 9, 2018

Exercise 1. For each of the following matrices, describe in geometric terms the real eigenspaces (if any) and their associated eigenvalues. Do not compute the matrices.

(a) The matrix induced by the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ which reflects each vector across the z-axis.

(b) The matrix induced by the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates each vector by $\pi/4$ radians counterclockwise.

For part (b), discuss with your group how complex eigenvectors and eigenvalues may be involved.

Exercise 2. Let T be defined by $T(\mathbf{x}) = A\mathbf{x}$. Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ with the property that $[T]_{\mathcal{B}}$ is diagonal.

$$A = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}$$

Exercise 3. (a) Find eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$ of the following matrix:

$$\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

(b) Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix has the form PCP^{-1}.

Exercise 4. The following matrix is the matrix for a composition of a rotation and a scaling. Give the angle ϕ of rotation and the scalar factor r.

$$\begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix}$$

Exercise 5. True or false? Justify please! Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in $\mathbb{R}^n$.

(a) $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$

(b) $\text{dist}(\mathbf{u}, \mathbf{v}) + \text{dist}(\mathbf{v}, \mathbf{w}) = \text{dist}(\mathbf{u}, \mathbf{w})$

Exercise 6. Show that if A is a real $n \times n$ matrix with $A^T = A$ and $A\mathbf{x} = \lambda\mathbf{x}$ for some nonzero vector $\mathbf{x}$ in $\mathbb{C}^n$, then λ is real. (This is a challenge! See 23 and 24 in section 5.5 for hints!)