Worksheet 12

Sections 306 and 310
MATH 54

October 2, 2018

Exercise 1. Mark each statement True or False. Justify each answer.

(a) The columns of $P_{\mathcal{C}\leftarrow\mathcal{B}}$ are linearly independent.

(b) If $V = \mathbb{R}^2$, $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$, and $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}\}$, then row reduction of $[\mathbf{c_1} c_2 b_1 b_2]$ to $[IP]$ yields a matrix P that satisfies $[\mathbf{x}]_{\mathcal{B}} = P[\mathbf{x}]_C$ for all $\mathbf{x}$ in V.

Exercise 2. In $\mathbb{P}_2$, find the change-of-coordinates matrix from the bases $\mathcal{B} = \{1 - 3t^2, 2 + t - 5t^2, 1 + 2t\}$ to the standard basis. Then write t_2 as a linear combination of the polynomials in $\mathcal{B}$.

Exercise 3. (a) As a group, discuss the definitions of eigenvector and eigenvalue. Draw pictures!

(b) Is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? If so, find the eigenvalue.

(c) Is $\lambda = 3$ an eigenvalue of $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Exercise 4. Find a basis for the eigenspace corresponding to each listed eigenvalue.

$$A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} \qquad \lambda = 4$$

Exercise 5. Explain why a 2×2 matrix can have at most two distinct eigenvalues. Explain why an $n \times n$ matrix can have at most n distinct eigenvalues.

Exercise 6. Find the characteristic polynomial and the eigenvalues for the matrix.

$$\begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}$$