Worksheet 11

Sections 306 and 310
MATH 54

September 27, 2018

Exercise 1. If a 6×3 matrix has rank 3, find dim Nul A, dim Row A, and rank A^T.

Exercise 2. If A is a 6×4 matrix, what is the smallest possible dimension of Nul A?

Exercise 3. Let $\mathcal{A} = \{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ and $\mathcal{D} = \{\mathbf{d_1}, \mathbf{d_2}, \mathbf{d_3}\}$ be bases for a vector space V, and let $P = [[\mathbf{d_1}]_{\mathcal{A}}, [\mathbf{d_2}]_{\mathcal{A}}, [\mathbf{d_3}]_{\mathcal{A}}]$. Which of the following equations is true for all $\mathbf{x}$ in V?

(a) $[\mathbf{x}]_{\mathcal{A}} = P[\mathbf{x}]_{\mathcal{D}}$

(b) $[\mathbf{x}]_{\mathcal{D}} = P[\mathbf{x}]_{\mathcal{A}}$

Exercise 4. Let $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ and $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}\}$ be bases for $\mathbb{R}^2$. Compute the change of coordinate matrix from $\mathcal{C}$ to $\mathcal{B}$.

$$b_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}, b_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, c_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, c_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$