Worksheet 1 Solutions

Sections 306 and 310
MATH 54

August 23, 2018

Exercise 1. Which of the following matrices are in row echelon form? For each matrix, write a corresponding system of linear equations.

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 3 & 2 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

This violates row-echelon form because the leading term of the second row is to the left of the leading term of the first row.

Exercise 2. The following three matrices are already in row echelon form. Which represent a consistent system of equations?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$0 \neq 4$ so this is not consistent

This actually has infinitely many solutions :)

$x = 0$
$y = 0$
$z = 1$

$w + x + z = 1$
$2x + 2z = 2$
$3z = 3$
$0 = 4$

$x + y = 1$
$z = 1$
$0 = 0.$

Exercise 3. Put the following in row echelon form.

$$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix} \qquad \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Note! Row echelon form is not unique, so there are many possible answers. I will just give 1:

a) $\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix} \xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ This is already in row-echelon form :)

Exercise 4. Describe the possible echelon fomrs of a nonzero 3×2 matrix. Use the symbols $\square$, $*$, and 0, where $\square$ means a nonzero number and $*$ means any number.

$$\begin{bmatrix} \blacksquare & \star \\ 0 & \blacksquare \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} \blacksquare & \star \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Exercise 5. Determine the values of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & h & 4 \end{bmatrix}$$

We first put this into row echelon form so that we can better see what's going on.

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & h & 4 \end{bmatrix} \xrightarrow{\frac{1}{3}R_1 \to R_1} \begin{bmatrix} 1 & 2 & 2 \\ 1 & h & 4 \end{bmatrix} \xrightarrow{-R_1+R_2 \to R_2}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & h-2 & 2 \end{bmatrix}$$

This is the augmented matrix for

$$x + 2y = 2$$
$$(h-2)y = 2$$

This has a solution iff $h-2 \neq 0 \iff h \neq 2.$