Worksheet 9

Sections 207 and 219
MATH 54

Febrary 21, 2018

Exercise 1. Determine if the following sets of polynomials are subspaces of the space $\mathbb{P}_3$ of polynomials in t of degree at most 3.

a. All polymonials in $\mathbb{P}_3$ of the form $p(t) = at^2$, where a is in $\mathbb{R}$.

b. All polynomials in $\mathbb{P}_3$ of the form $a + t^2$, where a is in $\mathbb{R}$.

c. All polynomials p in $\mathbb{P}_3$ such that $p(0) = 0$.

Exercise 2. Let W be the union of the first and third quadrants of the plane. That is: let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$.

(a) If $\mathbf{u}$ is in W and c is any scalar, is $c\mathbf{u}$ in W? Why?

(b) Can you find specific vectors $\mathbf{u}$ and $\mathbf{v}$ in W such that their sum is not in W.

(c) Is W a vector space?

Exercise 3. For each of the following sets, either use an appropriate theorem to show that the given set is a vector space, or find an specific example to the contrary.

$$\left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 5r - 1 = s + 2t \right\}$$

$$\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{matrix} a + 3b = c \\ b + c + a = d \end{matrix} \right\}$$

Exercise 4. Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Is $\mathbf{w}$ in Col A? Is it in Nul A?

Exercise 5. True or false? Justify each answer:

(a) A null space is a vector space.

(b) The column space of an $m \times n$ matrix is in $\mathbb{R}^m$.

(c) Col A is the set of all solutions of $A\mathbf{x} = \mathbf{b}$.

(d) Nul A is the kernel of the mapping $\mathbf{x} \mapsto A\mathbf{x}$.

(e) The range of a linear transformation is a vector space.