

Worksheet 8

Sections 207 and 219
MATH 54

February 19, 2018

Exercise 1. Determine the values of s such that the system has a unique solution. Use Cramer's rule to describe the solutions in terms of s .

$$3sx_1 + 5x_2 = 3$$

$$12x_1 + 5sx_2 = 2$$

Exercise 2. Compute the adjugate of the given matrix, and use theorem 8 to give the inverse of the matrix:

$$\begin{bmatrix} 1 & 1 & 3 \\ -2 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Exercise 3. Suppose that all the entries in A are integers. Are the entries in A^{-1} necessarily integers? What if $\det(A) = 1$? Explain.

Exercise 4. Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1,3,0)$, $(-2,0,2)$, and $(-1,3,-1)$.

Exercise 5. Let S be the parallelogram determined one vertex at the origin and adjacent vertices at $(-2,3)$ and $(-2,5)$. Let $A = \begin{bmatrix} 6 & -3 \\ -3 & 2 \end{bmatrix}$. Compute the area of the image of S under the mapping $\mathbf{x} \mapsto A\mathbf{x}$. Try computing in two different ways!

Exercise 6. Find a basis of $\text{Nul}(A)$ by listing vectors that span the null space:

$$\begin{bmatrix} 1 & 6 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

How many entries do vectors in the null space have? How many entries do vectors in the column space have? What is the dimension of each space?