Worksheet 6

Sections 207 and 219
MATH 54

February 7, 2019

Exercise 1. Let A be an $n \times n$ matrix. Suppose the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution for some $\mathbf{b}$ in $\mathbb{R}^n$. Can the columns of A span $\mathbb{R}^n$?

Exercise 2. Determine which of the matrices are invertible. Justify your answers, but try using as few calculations as possible :)

$$\begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix} \qquad \begin{bmatrix} -7 & 0 & 4 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 5 & 9 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

Exercise 3. Is it possible for a 5×5 matrix to be invertible when its columns do not span $\mathbb{R}^5$? Why or why not?

Exercise 4. Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Is $\mathbf{w}$ in Col A? Is it in Nul A?

Exercise 5. Determine which of the following sets are bases for $\mathbb{R}^3$. Justify your answers.

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} -4 \\ -5 \\ 6 \end{bmatrix}$$

Discuss with your group: Do you think that a set of two vectors can form a basis for $\mathbb{R}^3$? Why or why not? (We will discuss the idea of dimension soon, get excited!!)

Exercise 6. Assume that A is row equivalent to B. Find bases for nul A and col A.

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & 3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercise 7. True or false? Give brief justifications.

(a) A linearly independent set in a subspace H is a basis for H.

(b) If a finite set S of nonzero vectors spans a vector space V, then some subsets of S is a basis of V.

(c) If B is an echelon form of a matrix A, the pivot columns of B form a basis of col A.