Worksheet 5

Sections 207 and 219
MATH 54

February 5, 2019

Exercise 1. Show that if the columns of B are linearly dependent, then so are the columns of AB.

Exercise 2. Find matrices A, B, C, such that $AB = AC$, yet $B \neq C$.

Exercise 3. Solve the system using matrix inverses!

$$8x_1 + 5x_2 = -9$$

$$-7x_1 - 5x_2 = 11$$

Exercise 4. Suppose $(B - C)D = 0$, where B and C are $m \times n$ matrices and D is an invertible $n \times n$ matrix. show that $B = C$. Is this necesarily true if D is not invertible?

Exercise 5. Explain why the columns of an $n \times n$ matrix A span $\mathbb{R}^n$ when A is invertible.

Exercise 6. In this exercise, we prove that the inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ exists if and only if $ad - bc \neq 0$. In the case that it does exist, $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

(a) Show that if $ad - bc = 0$, then the equation for $A\mathbf{x} = \mathbf{0}$ has more than one solution. Conclude that then A must not be invertible.

(b) Show that if $ad - bc \neq 0$, then the formula holds.