Worksheet 4

Sections 207 and 219
MATH 54

January 31, 2019

Exercise 1. For each pair $A, \mathbf{b}$, let T be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$. For each, find a vector whose image under T is $\mathbf{b}$. Is this vector unique?

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

Exercise 2. Describe geometrically what the following linear transformation T does. It may be helpful to plot a few points and their images!

$$T = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$$

Exercise 3. Let $\mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y_1} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$ and $\mathbf{y_2} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{e_1}$ to $\mathbf{y_1}$ and $\mathbf{e_2}$ to $\mathbf{y_2}$. What is the image of $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$?

Exercise 4. Show that $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$ is a linear transformation.

Exercise 5. Assume T is a linear transformation. Find the standard matrix of T.

- $T : \mathbb{R}^3 \to \mathbb{R}^2$, and $T(\mathbf{e_1}) = (1, 3)$, $T(\mathbf{e_2}) = (4, -7)$, $T(\mathbf{e_3}) = (-4, 5)$, where $\mathbf{e_1}$, $\mathbf{e_2}$, and $\mathbf{e_3}$ are the columns of the 3×3 identity matrix.
- $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the horizontal x_1- axis and then reflects points through the line $x_1 = x_2$.

- $T : \mathbb{R}^2 \to \mathbb{R}^3$ and $T(x_1, x_2) = (x_1 - x_2, -2x_1 + x_2, x_1)$.

As a group, choose one of these transformations and figure out if it is one-to-one and onto.