Worksheet 3

Sections 207 and 219
MATH 54

January 29, 2018

Exercise 1. Write the following vector equation as a matrix equation and also as a system of linear equations. Solve the system and express your answer in parametric vector form.

$$x_1 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Exercise 2. Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}$.

$$\mathbf{a_1} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \mathbf{a_2} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \mathbf{a_3} = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ 8 \end{bmatrix}$$

Exercise 3. Write the following products as linear combinations of the columns of the matrix. Use this to compute the product.

$$\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Exercise 4. Do the following vectors span $\mathbb{R}^3$?

$$\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$$

Exercise 5. Let A be a 3×2 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b}$ in $\mathbb{R}^3$. (i.e. you can always find a $\mathbf{b}$ such that the equation is inconsistent) Generalize your argument to the case of an arbitrary$A$ with more rows than columns.

Exercise 6. Describe all solutions of $A\mathbf{x} = \mathbf{0}$, for the following matrices. Express your answers in parametric vector form.

$$A = \begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

Exercise 7. Describe the solutions of the system given by the following augmented matrix. Express your answer in parametric vector form.

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{bmatrix}$$

Exercise 8. Determine if each set of vectors is linearly independent.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 343 \\ 454 \\ 55 \\ -45 \\ 67 \end{bmatrix}$$

Exercise 9. Determine the possible row echelon forms of a 2×2 matrix with linearly dependent columns.