Worksheet 24

Sections 207 and 219
MATH 54

April 30, 2019

Exercise 1. For the following function, determine the π-periodic extension, the odd 2π periodic extension, and the even 2π periodic extension. Sketch the graphs of each.

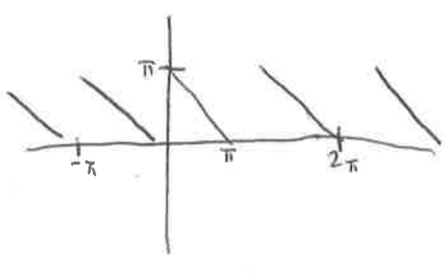
$$f(x) = \pi - x$$

I should have said that $0 < x < \pi$ — so $L = \pi$

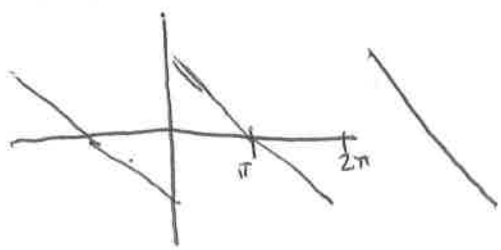
Exercise 2. For the f above, compute the fourier sine series. Sketch the graph of the fourier sine series (including filling in the points at the jump discontinuities). The sine series is related to which of the extensions discussed in problem 1?

Exercise 3. For the f above, compute the fourier cosine series. Sketch the graph of the fourier cosine series (including filling in the points at the jump discontinuities). The cosine series is related to which of the extensions discussed in problem 1?

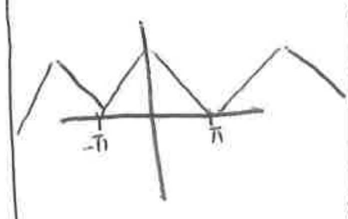
$\boxed{1}$ π-periodic extension:



2π odd



2π even



$\boxed{2}$ Our goal is to write $f(x)$ as $\sum_{n=1}^{\infty} b_n \sin(nx)$, ($\frac{n\pi x}{L} = nx$ in this case since $L = \pi$). The formula for the b_n is $\frac{2}{\pi}\int_0^{\pi} (\pi - x)\sin(nx)\,dx$.

We integrate by parts with: $u = \pi - x$, $du = \cancel{-x\,dx}$ $-dx$; $dv = \sin(nx)\,dx$, $v = -\frac{\cos(nx)}{n}$

So $b_n = \frac{2}{\pi}\left[uv\Big|_0^{\pi} - \int_0^{\pi} v\,du\right] =$

$\frac{2}{\pi}\left[(\pi - x)\left(-\frac{\cos(nx)}{n}\right)\Big|_0^{\pi} - \int_0^{\pi} \frac{\cos(nx)}{n}\,dx\right] =$

$\frac{2}{\pi}\left[\left[0 - \pi\left(-\frac{1}{n}\right)\right] - \left[\frac{\sin(nx)}{n^2}\Big|_0^{\pi}\right]\right] =$

$\frac{2}{\pi}\left[\left(\frac{\pi}{n}\right) - 0\right] = \frac{2}{n}$. So $f(x) \sim \sum_{n=1}^{\infty} \frac{2}{n}\sin(nx)$

$\boxed{3}$ similar, the answer you should get is $\frac{\pi}{2} + \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\cos(2k-1)x$