Worksheet 21

Sections 207 and 219
MATH 54

April 18, 2019

Exercise 1. Consider $\mathbf{x}'(t) = A\mathbf{x}(t)$, $t \geq 0$, where $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$.

(a) Show that the eigenvalues are -1,-3 and $\mathbf{u_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{u_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are the corresponding eigenvectors.

(b) Sketch the tractory of the solution having initial vector $\mathbf{x}(0) = \mathbf{u_1}$.

(c) Sketch the trajectory of the solution having initial vector $\mathbf{x}(0) = -\mathbf{u_2}$.

(d) Sketch the trajectory of the solution having initial vector $\mathbf{x}(0) = \mathbf{u_1} - \mathbf{u_2}$.

Exercise 2. Give a general solution to:

$$\mathbf{x}'(t) = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$$

Exercise 3. Use the substitutions $x_1 = y$, $x_2 = y'$ to convert $ay'' + by' + cy = 0$ into a normal system. Show that the characteristic equation of the system is the same as the aux equation for the original equation.

Exercise 4. Find a general solution of the system $\mathbf{x}'(t) = A\mathbf{x}(t)$ for the given matrix A. You can use the fact that the eigenvalues of A are 2, $2+i$, and $2-i$.

$$A = \begin{bmatrix} 5 & -5 & -5 \\ -1 & 4 & 2 \\ 3 & -5 & -3 \end{bmatrix}$$