Worksheet 18

Sections 207 and 219
MATH 54

April 9

Exercise 1. Find a general solution to the given differential equations:

(a) $y'' - y' - 2y = 0$

(b) $y'' - 5y' + 6y = 0$

(c) $4y'' - 4y + y = 0$

Exercise 2. Solve the initial given value problems:

(a) $y'' + y' = 0$; $y(0) = 2$; $y'(0) = 1$

(b) $y'' - 4y' + 4y$; $y(1) = 1$; $y'(1) = 1$

Exercise 3. (a) With your group, reread definition 1 in this section (restated below):

A pair of functions $y_1(t)$ and $y_2(t)$ is said to be linearly independent on an interval I if and only if neither of them is a constant multiple of the other on all of I.

(b) Are $y_1(t) = e^{3t}$ and $y_2(t) = e^{-4t}$ linearly independent?

(c) Are $y_1 = \tan^2(t) - \sec^2(t)$ and $y_2(t) = 3$ linearly independent?

Exercise 4. (a) Explain why two functions are linearly dependent on an interval I if and only if there exist constants c_1 and c_2, not both zero, such that

$$c_1 y_1(t) + c_2 y_2(t) = 0$$

for all t in I.

(b) Discuss with your group how this connects to the idea of linear independence that we discussed in the linear algebra section of the course.

(c) Expand the defintion of linear independent functions to apply to sets of 3 or more functions.

Exercise 5. We are starting a new section of this course, so there may be vocabulary that you have not seen before. In the context of this chapter, make sure you know what each of the words/phrases means in your own words:

(a) an interval I

(b) general solution

(c) initial value problem

(d) uniqueness

Exercise 6. Find a general solution to the given differential equations:

(a) $y'' + y = 0$

(b) $y'' - 10y' + 26y = 0$

(c) $y'' - 4y' + 7y = 0$

Exercise 7. Find a general solution to the following higher-order equation:

$$y''' - y'' + y' + 3y = 0$$

Exercise 8. Prove the sum of angles formula for the sine function by following these steps. Let x be a fixed constant.

(a) Let $f(t) = \sin(x + t)$. Show that $f''(t) + f(t) = 0$, $f(0) = \sin x$, and $f'(0) = \cos(x)$.

(b) Use the auxiliary techniqe to solve the initial value problem $y'' + y = 0$, $y(0) = \sin(x)$, and $y'(0) = \cos(x)$.

(c) By uniqueness, the solution in part (b) is the same as $f(t)$ from part (a). Write this equality, this should be the standard sum of angles formula for sin(x+t).