Worksheet 16

Sections 207 and 219
MATH 54

π day, 2019

Exercise 1. Show that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ form an orthoganal basis for $\mathbb{R}^3$. Then express $\mathbf{x}$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

Exercise 2. Write $\mathbf{y}$ as the sum of two orthogonal vectors, one in span$\{\mathbf{u}\}$ and one orthogonal to it.

$$\mathbf{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Compute the distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and the origin.

Exercise 3. True and false! Justify your answers!

(a) If A is an $n \times n$ matrix with orthogonal columns, then it is invertible.

(b) If a set $\{\mathbf{u_1}, \ldots \mathbf{u_p}\}$ has the property that $\mathbf{u_i} \cdot \mathbf{u_j} = 0$ whenever $i \neq j$ then S is an orthnormal set.

(c) If c is not 0, then the orthogaonl projection of $\mathbf{y}$ onto a vector $\mathbf{u}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{u}$.

Exercise 4. Let W be the subspace spanned by the $\mathbf{v}'s$ and write $\mathbf{y}$ as a sum of a vector in W and a vector orthogonal to W.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

What is the closest point in W to $\mathbf{y}$?

Exercise 5. Find the orthogonal complement of W, where W is the span of the following two vectors:

$$\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

Exercise 6. Without looking at the proof in the book, show that a set of nonzero orthogonal vectors is linearly independent.