Worksheet 14

Sections 207 and 219
MATH 54

March 12, 2019

Exercise 1. (a) Find eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$ of the following matrix:

$$\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

(b) Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix has the form PCP^{-1}.

Exercise 2. The following matrix is the matrix for a composition of a rotation and a scaling. Give the angle ϕ of rotation and the scalar factor r.

$$\begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix}$$

Exercise 3. True or false? Justify please! Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in $\mathbb{R}^n$.

(a) $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$

(b) dist$(\mathbf{u}, \mathbf{v})$+dist$(\mathbf{v}, \mathbf{w})$ =dist$(\mathbf{u}, \mathbf{w})$

Exercise 4. Find a unit vector in the direction of the given vector. Draw a picture of what an orthogonal vector would look like.

$$\begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$$

Exercise 5. True and false! Justify your answers!

(a) For any scalar c, $||c|| = c||||$.

(b) If is orthoganal to every vector in a subspace W, then is in $W^{\perp}$.

(c) If $||\mathbf{u}||^2 + ||||^2 = ||\mathbf{u} + ||^2$, then $\mathbf{u}$ and are orthogonal.

(d) For an $m \times n$ matrix A, vectors in nul A are orthogonal to vectors in row A.

Exercise 6. For what values of b is the following matrix diagonalizable?

$$\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$