Worksheet 13

Sections 207 and 219
MATH 54

March 7, 2019

Exercise 1. Mark each statement True or False. Justify each answer. For these problems, A, B are $n \times n$ matrices.

(a) If A, B are row equivalent, then they have the same eigenvalues.

(b) If A has n eigenvectors, A is diagonalizable.

(c) If A has n distinct eigenvalues, it is diagonalizable.

(d) If A is diagonalizable, then A has n distinct eigenvalues

Exercise 2. Let T be defined by $T(\mathbf{x}) = A\mathbf{x}$. Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ with the property that $[T]_{\mathcal{B}}$ is diagonal.

$$A = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}$$

Exercise 3. (a) As a group, discuss why it is useful to be able to diagonalize a matrix!

(b) If possible, diagonalize the following matrix:

$$\begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}$$

Exercise 4. The eigenvalues of A are 2 and 8. Use this information to diagonalize A:

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

Exercise 5. Let $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$ be a bisis for a vector space V and $T : V \to \mathbb{R}^2$ be a linear transformation such that:

$$T(x_1\mathbf{b_1} + x_2\mathbf{b_2} + x_3\mathbf{b_3}) = \begin{bmatrix} 2x_1 - 4x_2 + 5x_3 \\ -x_2 + 3x_3 \end{bmatrix}$$

Find the matrix for T relative to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$.