Worksheet 12

Sections 207 and 319
MATH 54

March 5, 2018

Exercise 1. (a) As a group, discuss the definitions of eigenvector and eigenvalue. Draw pictures!

(b) Is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? If so, find the eigenvalue.

(c) Is $\lambda = 3$ an eigenvalue of $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Exercise 2. Find a basis for the eigenspace corresponding to each listed eigenvalue.

$$A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} \qquad \lambda = 4$$

Exercise 3. Explain why a 2×2 matrix can have at most two distinct eigenvalues. Explain why an $n \times n$ matrix can have at most n distinct eigenvalues.

Exercise 4. Find the characteristic polynomial and the eigenvalues for the matrix.

$$\begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}$$

Exercise 5. For each of the following matrices, describe in geometric terms the real eigenspaces (if any) and their associated eigenvalues. Do not compute the matrices.

(a) The matrix induced by the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ which reflects each vector across the z-axis.

(b) The matrix induced by the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates each vector by $\pi/4$ radians counterclockwise.