Worksheet 11

Sections 207 and 219
MATH 54

February 28, 2018

Exercise 1. Find the vector $\mathbf{x}$ determined by the given coordinate vector $[\mathbf{x}]_\beta$ and the given basis β.

$$\beta = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\} \qquad [\mathbf{x}]_\beta = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

Exercise 2. Find the coordinate vector $[\mathbf{x}]_\beta$ of $\mathbf{x}$ relative to the given basis β.

$$\beta = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\} \qquad \mathbf{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Exercise 3. Let $\mathcal{A} = \{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ and $\mathcal{D} = \{\mathbf{d_1}, \mathbf{d_2}, \mathbf{d_3}\}$ be bases for a vector space V, and let $P = [[\mathbf{d_1}]_\mathcal{A}, [\mathbf{d_2}]_\mathcal{A}, [\mathbf{d_3}]_\mathcal{A}]$. Which of the following equations is true for all $\mathbf{x}$ in V?

(a) $[\mathbf{x}]_\mathcal{A} = P[\mathbf{x}]_\mathcal{D}$

(b) $[\mathbf{x}]_\mathcal{D} = P[\mathbf{x}]_\mathcal{A}$

Exercise 4. Let $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ and $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}\}$ be bases for $\mathbb{R}^2$. Compute the change of coordinate matrix from $\mathcal{C}$ to $\mathcal{B}$.

$$b_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}, b_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, c_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, c_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Exercise 5. Mark each statement True or False. Justify each answer.

(a) The columns of $P_{\mathcal{C} \leftarrow \mathcal{B}}$ are linearly independent.

(b) If $V = \mathbb{R}^2$, $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$, and $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}\}$, then row reduction of $[\mathbf{c_1} c_2 b_1 b_2]$ to $[IP]$ yields a matrix P that satisfies $[\mathbf{x}]_\mathcal{B} = P[\mathbf{x}]_C$ for all $\mathbf{x}$ in V.

Exercise 6. In $\mathbb{P}_2$, find the change-of-coordinates matrix from the standared basis to the basis $\mathcal{B} = \{1 - 3t^2, 2 + t - 5t^2, 1 + 2t\}$. Then write t^2 as a linear combination of the polynomials in $\mathcal{B}$.