Worksheet 10

Sections 207 and 219
MATH 54

February 26, 2019

Exercise 1. Assume that A is row equivalent to B. Find a basis for the space spanned by the columns of A.

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & 3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercise 2. True or false? Give brief justifications.

(a) A linearly independent set in a subspace H is a basis for H.

(b) If a finite set S of nonzero vectors spans a vector space V, then some subsets of S is a basis of V.

(c) If B is an echelon form of a matrix A, the pivot columns of B for a basis of col A.

(d) Every plane in $\mathbb{R}^3$ is isomorphic to $\mathbb{R}^2$.

(e) The vector spaces $\mathbb{P}_2$ and $\mathbb{R}^3$ are isomorphic.

Exercise 3. Find a basis for the set of vectors in $\mathbb{R}^3$ on the plane $x + 2y + z = 0$.

Exercise 4. Using facts that we learned in chapter 1 and 2, explain why a basis of $\mathbb{R}^n$ must contain exactly n vectors.

Exercise 5. Let $T : V \to W$ be a linear transformation. Shot that if $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ is linearly dependent V, then $\{T(\mathbf{v_1}), \ldots, T(\mathbf{v_p})\}$ is linearly dependent in W. Use this to show that if $\{T(\mathbf{v_1}), \ldots, T(\mathbf{v_p})\}$ is linearly independent in W, then $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ is linearly independent in V.