Worksheet 1

Sections 207 and 219

MATH 54

Jan 21, 2019

Exercise 1. For each augmented matrix, write a corresponding system of linear equations. Can you tell (without doing any calculations) that one of these systems has no solutions?

$$
\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\qquad
\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 3 & 2 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}
\qquad
\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}
$$

Exercise 2. Write each of the following systems as an augmented matrix. Then, solve each system.

- $x_1 + 5x_2 = 3, \quad x_1 - x_2 = -3$

- $x - 2y = 4, \quad -3x + 6y = -12$

- $x - 2y = 4, \quad -3x + 6y = 5$

- $x_1 - 3x_2 = 5, \quad -x_1 + x_2 + 5x_3 = 2, \quad x_2 + x_3 = 0$

Exercise 3. If possible, compute each of $3C - E$, CB, EB. If any of these computations are impossible, briefly explain why.

$$
B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}
\qquad
C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}
\qquad
E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}
$$

Exercise 4. If a matrix B is 5×3 and the product AB is 2×3, what is the size of A? (For an $m \times n$ matrix, m is the number of rows and n is the number of columns.)

Exercise 5. (bonus problem!) Show that the following equation holds:

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$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} a \\ d \\ g \end{bmatrix} + x_2 \begin{bmatrix} b \\ e \\ h \end{bmatrix} + x_3 \begin{bmatrix} c \\ f \\ i \end{bmatrix}$$

The fact that multiplying a matrix by a vector gives a weighted sum of the columns of the matrix will be useful later! Don't worry too much if this problem doesn't make too much sense right now.