Worksheet 4.1 and 4.2

Max's Lecture
MATH 54

July 2, 2019

Exercise A (from Charles). Let a, b, c be integers. If $a|bc$, is it the case that a must divide b or c?

Exercise B(from Ritvik). Prove that if a is any integer other than 0, can you think of a number that divides a? Can you think of a number that a must divide?

Exercise C. What are the quotient and remainder when 11 is divided by 4? What if -25 is divided by 4?

Exercise D. 4.1.21,22 Let m be a positive integer. Prove the following:

1. Let m be a positive integer. Show that $a \equiv b(\mod m)$ if $a \mod m = b \mod m$.
2. Show that $a \mod m = b \mod m$ if $a \equiv b(\mod m)$.

Together, these two prove Theroem 2 in section 4.1

Exercise E (4.1.45). Show that if m is a positive integer of the form $4k + 3$ for some nonnegative integer k, then m is not the sum of the squares of two integers.

Exercise F. Are the following true? Prove or disprove. In all cases, a, b, c, d are integers, and m is a positive integer.

1. If $ac \equiv bc(\mod m)$ then $a \equiv b(\mod m)$.
2. If $a \equiv b(\mod m)$ and $c \equiv d(\mod m)$, then $a^c \equiv b^d(\mod m)$.

Exercise G. Try doing the following computations without using a calculator:

1. 6^{81} **mod** 7
2. 3^{18} **mod** 5

Exercise H (from charles). 1. Express 74 in base 2. Express 27 in hexidecimal.

2. Convert the binary number 10101 to base 4. Do the same for base 8. Can you guess a pattern?

Exercise I. Suppose that an integer is expressed in the standard decimal notation. How can you tell whether the number is divisible by 3? Why does your rule work?