

Worksheet 2.5 and 4.1

Max's Lecture
MATH 54

July 2, 2019

Exercise A (Charles worksheets). Show that any subset of a countable set is countable.

Solution

Q.

This is on a previous worksheet

Exercise B. (Ritviks worksheet) Find the cardinality of each of the following. Explain.

1. The integers less than 10
2. The integers with absolute value less than 50.
3. The rational numbers
4. The real numbers between 0 and 2.
5. The set $A \times \mathbb{Z}^+$ where A is the set $\{1, 2\}$.

This is on a previous worksheet.

Exercise C (from discussion in textbook). Consider Hilbert's Grand Hotel, which has countably infinite rooms. Suppose the hotel is full. If another guest shows up, can you accommodate them? If so, how would you do this?

Yes, we can accommodate them.

We tell every guest to move from room n (where n is the room they are currently staying in) to room $n+1$.

This makes it so room 1 is ~~open~~ open and available for the new guest.

Exercise D (from Charles). Let a, b, c be integers. If $a|bc$, is it the case that a must divide b or c ?

No! For example, $12 = 3 \cdot 4$.

$6|12$, but $6 \nmid 3$ and $6 \nmid 4$.

↙ Sorry, the preface
"prove that" was left
here by mistake.

Exercise E(from Ritvik). ~~Prove that~~ if a is any integer other than 0, can you think of a number that divides a ? Can you think of a number that a must divide?

For question 1: 1 must divide a .
For question 2: a must divide 0.

Exercise F. What are the quotient and remainder when 11 is divided by 4? What if -25 is divided by 4?

$$11 = 4 \cdot 2 + 3 \quad q=2 \text{ and } r=3$$

$$-25 = 4 \cdot 7 + 3 \quad q=7 \text{ and } r=3$$

Exercise Challenge problem! Let A be a nonempty set. Show that there is no surjection from A to its power set.

For sake of time, I'm not posting solutions to challenge problems. If you do have questions though, let me know!