Worksheet 2.5 and 4.1

Max's Lecture
MATH 54

July 2, 2019

Exercise A (Charles worksheets). Show that any subset of a countable set is countable.

Exercise B. (Ritviks worksheet) Find the cardinality of each of the following. Explain.

1. The integers less than 10

2. The integers with absolute value less than 50.

3. The rational numbers

4. The real numbers between 0 and 2.

5. The set $A \times \mathbb{Z}^+$ where A is the set $\{1, 2\}$.

Exercise C (from discussion in textbook). Consider Hilbert's Grand Hotel, which has countably infinite rooms. Suppose the hotel is full. If another guest shows up, can you accomodate them? If so, how would you do this?

Exercise D (from Charles). Let a, b, c be integers. If $a|bc$, is it the case that a must divide b or c?

Exercise E(from Ritvik). Prove that if a is any integer other than 0, can you think of a number that divides a? Can you think of a number that a must divide?

Exercise F. What are the quotient and remainder when 11 is divided by 4? What if -25 is divided by 4?

Exercise Challenge problem!. Let A be a nonempty set. Show that there is no surjection from A to its power set.