Worksheet 1.7-1.8

Max's Lecture
MATH 54

June 26, 2019

Exercise A (Charles wang's worksheet archive, 1.7.25). Prove the following by contradiction

1. $\sqrt[3]{2}$ is irrational.
2. At least 10 of any 64 days chosen must fall on the same day of the week.

Exercise B (1.8.33 and Charles's worksheet repository). Prove each of the proofs by casework:

1. Show that there are no solutions in positive intergers x and y to $x^4 + y^4 = 625$. (Hint: Which cases can you throw out right away?)
2. Prove that if the remainder when dividing n by 3 is 2, that n is not a square. (Hint: Try doing this by contraposition. For another hint, see me!)

Exercise C . Your friend shows you a proof of $|xy| = |x||y|$ for all real x and y. They prove this using the following 4 cases:

1. x, y both nonnegative
2. x nonnegative and y negative
3. x negative and y nonnegative
4. x, y both negative

How could we use the idea of WLOG to shorten the proof.

Exercise D (1.8.21). Show that if n is an odd integer, then there is a unique integer k such that n is the sum of $k-2$ and $k+3$.

Exercise E. (Example in book, and Ritviks worksheets.) Use these series of problems to practice the process of mathematical exploration! Have fun!

The standard checkerboard is an grid that is 8 squares by 8 squares. A domino is a piece that is 2 squares by 1 square. We say that a board is tiled by dominoes when all the squares are covered with no overlapping dominoes and no dominoes overhanging the edge?

1. Can we tile the standard checkerboard using the dominoes?
2. Try to tile a board obtained by removing one of the four corner squares of a standard checkerboard. Make a conjecture as to whether this is possible.
3. Prove your conjecture.
4. Try to tile a board obtained by removing two opposite corners of the standard checkerboard. Make a conjecture as to whether this is possible.
5. Prove your conjecture. (This is tricky! Ask me for a hint!)
6. Can you use dominoes to tile a standard checkerboard board with 2 adjacent corners removed? What about all 4 corners removed?
7. Prove or disprove that you can use dominoes to tile any rectangular checkerboard with an even number of squares i.e. an m by n board with mn (the number of squares) being even.