Worksheet 1.6-1.7

Max's Lecture
MATH 54

June 26, 2019

Exercise A (1.6.3,1.6.7). What rule of inference is used in each of these argments?

1. Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
2. Jerry is a mathematics major and a computer science major. THerefore, Jerry is a mathematics major.
3. If it is rainy, then the pool will be closed. It is rainy. Therefore the pool is closed.
4. If it snows today, the university will close. The university is not closed today. Therefore it did not snow today.
5. If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, I will sunburn. Therefore, if I go swimming, then I will sunburn.
6. All men are mortal. Socrates is a man. Therefore, Socratese is mortal.

Exercise B (Example from book). Show that the premises: " If you send me an e-mail, then I will finish writing the program," "If you do not send me an e-mail, then I will go to sleep early,", and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed".

Exercise C (1.6.19). Determine whether each of these arguments is valid. If not, what logical error occurs?

1. If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose $n^2 > 1$. Then $n > 1$.
2. If n is a real number with $n > 3$, then $n^2 > 9$. Suppose $n^2 \leq 9$. Then $n \leq 3$.
3. If n is a real number with $n > 2$, then $n^2 > 4$. Supose $n \leq 2$. Then $n^2 \leq 4$.

Exercise D (Rosen 1.7.1,1.7.7. Find a direct proof of the following:

1. The sum of two odd integers is even.
2. Every odd integer is the difference of two squares. (Hint! Find the difference of k and $k+1$)

Exercise E. (Rosen 1.7.19) Show that if n is an integer and $n^3 + 5$ is odd, then n is even. Try doing this in two ways, using a proof by contraposition and a proof by contradiction.

Exercise F (Example in book). The following is a famous proof that 1=2. Where is the mistake?

Suppose that $a = b$. We then multiply both sides by a to get $a^2 = ab$. We then subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Factoring both sides, we get $(a-b)(a+b) = b(a-b)$. Then divide both sides by $(a-b)$ to get $(a+b) = b$. Since $a = b$, this is the same as $2b = b$. Dividing by b, we get $2 = 1$.

Exercise Challenge Problem 1 (rosen 1.7.44). Prove that the following for statements about an integer n are equivalent.

1. n^2 is odd
2. $1 - n$ is even
3. n^3 is odd
4. $n^2 + 1$ is even