Worksheet 10.1

Max's Lecture
MATH 55

August 5, 2019

Exercise A. Decide whether you would use a graph or a directed graph to model the following scenarios. Then draw the graph or digraph.

1. Max and Gus are friends on Facebook. Lia and Max are friends on facebook.
2. Max, Gus, and Zyk all follow each other on instagram. Max follows their favorite band on instagram but their favorite band does not follow them back. :(

1. undirected graph

directed graph.

Exercise B. 1. How many simple graphs are there with n vertices?

2. How many relations are there on a set with n elements such the relation is both symmetric and reflexive?
3. Did you get the same answer for the first two parts of this question? Can you explain why or why not? (no formal proof necesary)

1. $2^{\binom{n}{2}}$ There are $\binom{n}{2}$ possible edges since each edge is uniquely determined by 2 vertices. We then for each possible edge, choose whether or not it exists.

2. $2^{\binom{n}{2}}$ For each subset $\{a,b\}$, where $a \neq b$, we choose whether or not (a,b) (and thus (b,a) because of symmetry) are in the relation. There are $\binom{n}{2}$ of these subsets. We already know that all pairs of the form (a,a) are in the relation.
So there are $2^{\binom{n}{2}}$ such relations.

Exercise C. Use the handshaking lemma to prove the following theorem: An undirected graph has an even number of vertices of odd degree.

By the handshaking lemma, ~~there~~.

$$2e = \sum_{v \in V} \deg(v) = \sum_{\substack{v \text{ has} \\ \text{odd} \\ \text{degree}}} \deg(v) + \sum_{\substack{v \text{ has} \\ \text{even} \\ \text{degree}}} \deg(v).$$

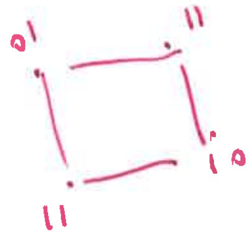
These two quantities are even, so in order for the equality to hold, $\sum_{\substack{v \text{ has} \\ \text{odd} \\ \text{degree}}} \deg(v)$

must be even. All the summands in this sum are odd, so to make this term even there has to be an even # of them.

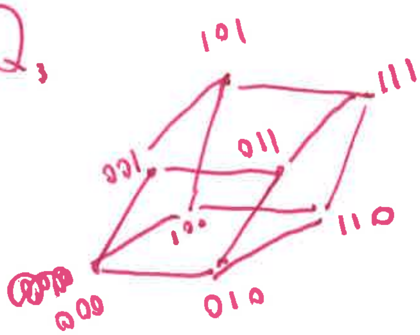
Exercise D. Let Q_n, called the n-cube graph, be the graph where each vertex is a bit string of length n, and two vertices are adjacent if and only if the bit strings that they represent differ in exactly one bit position. Draw Q_1, Q_2,Q_3. Why do you think these graphs are called cubes?

Q_1:

Q_2:



Q_3



They are the "skeletons" of n-dimensional cubes.

Exercise E. Determine for which n the following types of graph are bipartite? For the cases in which the graphs are bipartite, describe you would would go about coloring the vertices?

1. K_n Bipartite for $n = 1, 2$.
2. C_n Bipartite for even n but not odd.
3. W_n Never bipartite.
4. Q_n Always bipartite.

For Q_n, we can color all strings with an odd number of 1's blue, and all strings with an even number of 1's red.