Worksheet 10.1

Max's Lecture MATH 55

August 5, 2019

Exercise A. Decide whether you would use a graph or a directed graph to model the following scenarios. Then draw the graph or digraph.

- 1. Max and Gus are friends on Facebook. Lia and Max are friends on facebook.
- 2. Max, Gus, and Zyk all follow each other on instagram. Max follows their favorite band on instagram but their favorite band does not follow them back. :(

Exercise B. 1. How many simple graphs are there with *n* vertices?

- 2. How many relations are there on a set with n elements such the relation is both symmetric and reflexive?
- 3. Did you get the same answer for the first two parts of this question? Can you explain why or why not? (no formal proof necesary)

Exercise C. Use the handshaking lemma to prove the following theorem: An undirected graph has an even number of vertices of odd degree.

Exercise D. Let Q_n , called the *n*-cube graph, be the graph where each vertex is a bit string of length n, and two vertices are adjacent if and only if the bit strings that they represent differ in exactly one bit position. Draw Q_1 , Q_2 , Q_3 . Why do you think these graphs are called cubes?

Exercise E. Determine for which n the following types of graph are bipartite? For the cases in which the graphs are bipartite, describe you would would go about coloring the vertices?

- 1. K_n
- 2. C_n
- 3. W_n
- 4. Q_n