Worksheet 9.6

Max's Lecture
MATH 55

July 31, 2019

Exercise A. For the two equivalence relations in the exercise above, describe all equivalence classes.

1. $a = b$ or $a = -b$
2. $x \equiv y \pmod 7$

On a previous worksheet.

partial order.

Exercise B. Determine whether the relation R on the set of all positive integers is an ~~equivalence relation~~, where $(x, y) \in R$ if and only if:

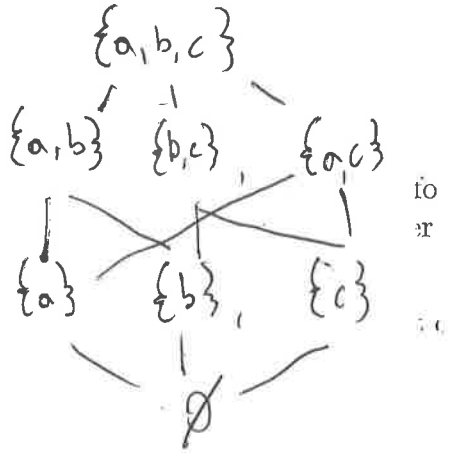
1. $x = y$ Yes!
2. $x < y$ No, not reflexive.
3. $x|y$ Yes!
4. $x \leq y$ Yes!

here, we assume it is finite.

Exercise C. For each of the following posets, let A be a subset of the elements. Give a description of the greatest lower bound of A and the least upper bound of A. It may help to sketch a Hasse Diagram!

1. S is the power set of a set of n elements, and the subsets in the power set are ordered by containment.
2. $(\mathbb{Z}^+, \leq)$
3. $(\mathbb{Z}^+, |)$

1. I will draw an example with 3 elements:



It turns out that the greatest lower bound of a subset of poset elements corresponds to the intersection of the underlying subsets.

(i.e. $\{b,c\}$ and $\{a,c\}$ have $\{c\}$ as their glb.)

The least upper bound corresponds to the union of the underlying subsets.

2. Here is a small part of the Hasse diagram:

The glb of A is the smallest element in the set.
The lub is the greatest element in the set.

3. We talked about this in class.

The glb is the gcd of the numbers in A.

The lub is the lcm of the numbers in A.