Worksheet 9.4-5

Max's Lecture
MATH 55

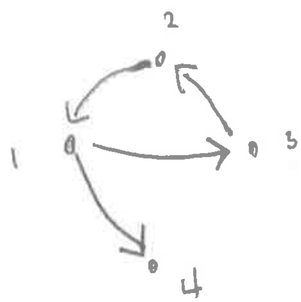
July 30, 2019

Exercise A. This isn't really an excercise, just a scenario that would take too long to write on the board. Consider a set of 5 cities. Consider the relation R on this set where $(a, b) \in R$ if and only if there is a direct telephone line connecting a and b. How can we determine whether two cities have (a possibly indirect) link?

The idea here is to motivate the idea of a transitive closure.

Exercise B. Consider the relation on $\{1, 2, 3, 4\}$ given by the pairs $(1, 3), (1, 4), (2, 1), (3, 2)$. Draw a digraph of this relation, and think about how you would write down the transitive closure.

$\emptyset$



The transitive closure is given by the following digraph:

Exercise C. Determine whether the relation R on the set of all integers is an equivalence relation, where $(x, y) \in R$ if and only if:

1. $x|y$ No, not symmetric.
2. $a = b$ or $a = -b$ Yes
3. $x = y + 1$ or $x = y - 1$ No, not reflexive or transitive.
4. $x \equiv y \pmod 7$ Yes.

Let me know if you want any more details.

Exercise D. For each of the two equivalence relations in the exercise above, describe all equivalence classes.

2. The equivalence classes are of the form:

$$\{0\}, \{1,-1\}, \{2,-2\}, \{3,-3\}, \ldots$$

3. The equivalence classes are of the form

$[0] = \{\ldots, -7, 0, 7, 14, \ldots\}$

$[1] = \{\ldots, -6, 1, 8, \ldots\}$

$[2] = \{\ldots, -5, 2, 9, \ldots\}$

$[3] = \{\ldots -4, 3, 10 \ldots\}$

$[4] = \{\ldots -3, 4, 11 \ldots\}$

$[5] = \{\ldots -2, 5, 12 \ldots\}$

$[6] = \{\ldots -1, 6, 13 \ldots\}$