Worksheet 1.1-1.5

Max's Lecture
MATH 54

June 25, 2019

Exercise A. Recall the de Morgen's laws:

$\neg(p \vee q) = \neg p \wedge \neg q$

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Convince yourself that these are true, either by using a truth table, a venn diagram, or some other argument.

Exercise B (1.2.39 from Rosen). A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying, and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.

Exercise C (from Ritviks worksheet archive). Find a compound proposition involving p, q, r that is true when exactly two of p, q, r are true and false otherwise.

Exercise D (Rosen 1.5.15. Determine the truth value of each of these statemennts if the domain for all variables consists of integers:

1. $\forall n(n^2 \geq 0)$
2. $\exists n(n^2 = 2)$
3. $\forall n(n^2 > n)$
4. $\exists n(n^2 < 0)$

Do your answers change if you let the domain be all real numbers? Two of these statements are negations of each others. Which ones?

Exercise E. Let $C(x)$ be the statement "x is a cat" and $F(x)$ be the statement "x is fluffy". Write the following statements in English:

1. $\exists x(C(x) \to F(x))$
2. $\exists x(C(x) \wedge F(x))$

Do these mean the same thing? Why or why not?

Exercise F (modified Rosen 1.5.10). Let $F(x, y)$ be the statement "x can fool y". Use quantifiers to express each of these statements:

1. Evelyn can fool everybody.
2. Everyone can be fooled somebody.
3. There is somebody who can fool everybody.
4. Challenge: Nancy can fool exactly two people.

Now negate each of these sentence both using logical notatation and then translate to English. Do not just say "it is not the case that blah blah blah".

Exercise G (Rosen 1.5.21). Use predicates, quantifiers, logical connectives, and mathematical operators to express the statement that every positive integer is the sum of the squares of four integers.

Exercise Challenge 1 (from Ritvik's worksheet archive). Consider an island with two kinds of inhabitants: Knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

1. A says at least one of us is a knave and B says nothing
2. A says "the two of us are both knights" and B says "A is a knave"

Exercise Challenge 2. It is a fact that every compound proposition (that perhaps uses implications, biconditonals, or logical operators that we have not learned yet) is logically equivalent to a compound proposition that uses only $\neg, \vee, \wedge$. Can you explain why this is true?