Worksheet 7.4 and 8.1

Max's Lecture
MATH 55

July 23, 2019

Exercise A. 1. What is the variance of the random variable X with $X(t) = 1$ if Bernoulli trial is a sucess and and $X(t) = 0$ is the Bernoulli trial is a failure, where p is the probability of success and q is the probability of failure?

2. Use the above to find the variance of the number of successes in scenario 1 of exercise D.

This was done on the previous worksheet.

Exercise B. You do 100 bernoulli trials (with the chance of success being 1/2 for each trial). The random variable X records the number of successes. What is the expected value of X? Give an **upper bound** on the probability that $X(s)$ differs from the expected value by at least 25?

(NOTE: the question was phrased incorrectly on the version that I handed out in class, I want an upper bound on the probability, not the actual probability)

This was done on the previous worksheet.

Exercise F. ind a recurrence relation for the following counting scenarios:

1. The number of bit strings of length n that have a pair of consecutive zeros.
2. The number of ways to tile an $n \times 2$ board using dominos.
3. C_n, where C_n is the number of ways to parenthesize the product of $n+1$ numbers, $x_0, x_1, \ldots x_n$, to specify the order of multiplication. For example, $C_3 = 5$ because we can write: $((x_0 \cdot x_1) \cdot x_2) \cdot x_3$, $(x_0 \cdot (x_1 \cdot x_2)) \cdot x_3$, $(x_0 \cdot x_1) \cdot (x_2 \cdot x_3)$, $x_0 \cdot ((x_1 \cdot x_2) \cdot x_3)$, and $x_0 \cdot (x_1 \cdot (x_2 \cdot x_3))$

1. Let a_n be # of such strings.
Suppose we want to build a string of length n. with 2 consecutive 0's. We can split this problem up into several cases (there is more than one way to do this.)

Case 1: The last bit is a 1. Then this last bit does not contribute to the 2 consecutive 0's, so they must appear in the first $n-1$ bits. There are $\underline{a_{n-1}}$ ways to choose these bits.

Case 2: The last bit is a 0. This may or may not contribute to the 2 consecutive 0's, so we consider 2 subcases.

Case 2.1: The second to last bit is a 1. so the last two bits are 10. Thus, the 2 consecutive 0's must appear in the first $n-2$ bits. There are $\underline{a_{n-2}}$ ways to choose this.

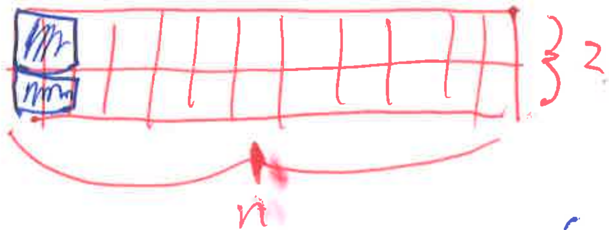
Case 2.2. The second to last bit is a 0, so the last two bits are 00. This already satisfies the condition that we have 2 consecutive 0's, so the first $n-2$ digits can be anything. There are 2^{n-2} ways to choose this.

Putting it all together, $a_n = a_{n-1} + a_{n-2} + 2^{n-2}$.

2. Let a_n be # of ways to tile an $(n \times 2)$ board.

We break this up into 2 cases:

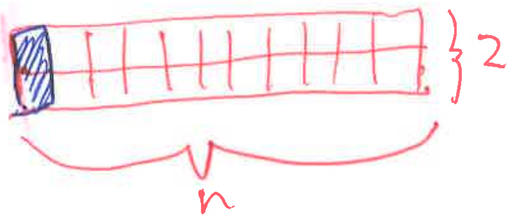
Case 1: There is a horizontal domino in the top left corner.



This forces there to be a horizontal domino below it.

We are left with an $(n-2) \times 2$ board, which can be tiled in a_{n-2} ways.

Case 2. There is a vertical domino in the top left corner.



We are left with an $(n-1) \times 2$ board, which can be tiled in a_{n-1} ways.

So putting everything together, we get $a_n = a_{n-1} + a_{n-2}$ (the same recurrence as fibonacci).

3.

One possible recurrence is

$$C_n = C_0 C_{n-1} + C_1 C_{n-2} + \dots + C_{n-2} C_1 + C_{n-1} C_0$$

An explanation is on page 532-533