

# Worksheet 7.4 and 8.1

Max's Lecture  
MATH 55

July 23, 2019

- Exercise A.**
1. What is the variance of the random variable  $X$  with  $X(t) = 1$  if Bernoulli trial is a success and  $X(t) = 0$  if the Bernoulli trial is a failure, where  $p$  is the probability of success and  $q$  is the probability of failure?
  2. Use the above to find the variance of the number of successes in scenario 1 of exercise D.

**Exercise B.** You do 100 bernoulli trials (with the chance of success being  $1/2$  for each trial). The random variable  $X$  records the number of successes. What is the expected value of  $X$ ? Give an **upper bound** on the probability that  $X(s)$  differs from the expected value by at least 25?

(NOTE: the question was phrased incorrectly on the version that I handed out in class, I want an upper bound on the probability, not the actual probability)

**Exercise F.** Find a recurrence relation for the following counting scenarios:

1. The number of bit strings of length  $n$  that have a pair of consecutive zeros.
2. The number of ways to tile an  $n \times 2$  board using dominos.
3.  $C_n$ , where  $C_n$  is the number of ways to parenthesize the product of  $n + 1$  numbers,  $x_0, x_1, \dots, x_n$ , to specify the order of multiplication. For example,  $C_3 = 5$  because we can write:  $((x_0 \cdot x_1) \cdot x_2) \cdot x_3$ ,  $(x_0 \cdot (x_1 \cdot x_2)) \cdot x_3$ ,  $(x_0 \cdot x_1) \cdot (x_2 \cdot x_3)$ ,  $x_0 \cdot ((x_1 \cdot x_2) \cdot x_3)$ , and  $x_0 \cdot (x_1 \cdot (x_2 \cdot x_3))$