Worksheet 7.4

Max's Lecture
MATH 55

July 22, 2019

Exercise A (example in book). Suppose 1 person in 100,000 has a particular rare disease for which their is a fairly accurate diagnostic test. This test is correct 99 percent of the time when given to a person who has the disease, and it is correct 99.5 percent of the time when given to a person at random who does not have the disease.

Given this information, can we find:

1. the probability that a person who tests positive for the disease has the disease?
2. the probability that a person who tests negative for the disease really does not have the disease?

Exercise B. Prove the theorem that expected value is linear, i.e. prove the following two facts if $X, X_1, X_2 \dots, X_n$ are random variables on the same sample space and a, b are real numbers:

1. $E(X_1 + \cdots + X_n) = E(X_1) + \dots E(X_n)$.
2. $E(aX + b) = aE(X) + b$.

Exercise C (example in book). Suppose you roll a pair of dice. Use the theorem in Exercise B to compute the expected value of their sum.

Exercise D.

1. Suppose you run n Bernoulli trials, where the probability of success is p. What is the expected value of the number of success?
2. Suppose you keep repeating a Bernoulli trial until you get a success, and then stop after the first success where the probability of success is p. What is the expected number of trials you will do?

Note: The first part has you compute the expected value of a binomial distribution, and the second has you compute the expected value of a geometric distribution.

Exercise E. 1. What is the variance of the random variable X with $X(t) = 1$ if Bernoulli trial is a sucess and and $X(t) = 0$ is the Bernoulli trial is a failure, where p is the probability of success and q is the probability of failure?

2. Use the above to find the variance of the number of successes in scenario 1 of exercise D.

Exercise F. You do 100 bernoulli trials (with the chance of success being 1/2 for each trial). The random variable X records the number of successes. What is the expected value of X? What is an upper bound of the probability that $X(s)$ differs from the expected value by at least 25?

NOTE: The problem was incorrect on the version of the worksheet that I handed out in class. I want an upper bound, not the actual probability.