Worksheet 6.5 and 7.1

Max's Lecture
MATH 55

July 18, 2019

Exercise A (from previous worksheet. Compute the following:

1. The number of different strings that can be made from rearranging the 8 letters in AARDVARK.
2. Ther are 5 different flavors of donuts. How many ways can I make an order of 8 donuts?
3. How many 8-letter strings can I make using the letters A,R,D,V,K
4. How many ways can you put r indistinguishable balls into n distinguisible boxes?

1. This is an example of a permutation in which some elements are indistinguishible.
   We have 8 letters total, with 3 A's, 2 R's and one of everything else. So there are $\frac{8!}{3!2!}$ ways to rearrange.

2. This is an example of a combination with repitition (we don't care about the ordering of the donuts, we just care about how many there are of each type.
   $n=5$, $r=8$, so there are $\binom{n+r-1}{r} = \binom{12}{8}$ ways.

3. This is an example of a permutation with repitition.
   For each slot _ _ _ _ _ _ _ _ we have 5 choices.
   So there are 5^8 ways.

4. For each ball, we have n options, and it doesn't matter the order in which the balls are arranged within each box. This is a combination with repitition (since you are allowed to put more than one ball in a box). So there are $\binom{n+r-1}{r}$ ways.

Exercise B (Various problems from 7.1). Compute the following probabilities:

1. What is the probability that the sum of the numbers on two dice is even when they are rolled?
2. What is the probability that a 5-card poker hand contains at least one ace?
3. What is the probability that a positive integer not exceeding 100 is divisible by 3?
4. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7?

$\boxed{1}$ The sum of the numbers is even either when both numbers are even (call this E_e) or when both numbers are odd (call this E_o) These events are disjoint, so we can add the probabilities together.

$$p(E_e) = \frac{\text{\# ways to have both numbers even}}{\text{total number of ways to roll 2 dice}}$$

$$= \frac{3\cdot 3}{6\cdot 6} = \frac{9}{36} = \frac{1}{4}.$$

$$p(E_o) = \frac{\text{\# ways to have both numbers odd}}{\text{total \# ways to roll 2 dice}}$$

$$= \frac{1}{4}.$$

Adding together, we get a probability of $\frac{1}{2}$

$\boxed{2}$ Let E be the event that the card has at least one ace. Then $\overline{E}$ is the event that it has no ace. Since $\overline{E}$ is easier (at least in my opinion) to calculate, we will do that first.

$$p(\overline{E}) = \frac{\text{\# ways to draw a hand without an ace}}{\text{\# ways to draw a hand}}$$

$$= \frac{\binom{48}{5}}{\binom{52}{5}}.$$

So $p(E) = 1 - p(\overline{E}) = 1 - \dfrac{\binom{48}{5}}{\binom{52}{5}}$

$\boxed{3}$ Let E be the set of numbers not exceeding 100 that are divisible by 3.
There are 33 of these, since $\lfloor \frac{100}{3} \rfloor = 33$.

Let our sample space, S, be the ~~numbers not exc~~ positive integers not exceeding 100. There are 100 of these.

So $p(E) = \frac{|E|}{|S|} = \frac{33}{100}$

$\boxed{4}$ Again, our sample space S is the set of positive integers not exceeding 100.

Let F be the subset of S that is divisible by 5. This has $\lfloor \frac{100}{5} \rfloor = 20$ elements.

~~We want to compute $p(E \cup F)$.~~
~~This is $p(E) + p(F) - p(E \cap F)$.~~

~~Since 5 and~~ Let E be the subset of S that is divisible by 7. This has $\lfloor \frac{100}{7} \rfloor = 14$ elements.

Our goal is to compute

$$p(E \cup F) = p(E) + p(F) - p(E \cap F).$$

We know $p(E) = \frac{14}{100}$, $p(F) = \frac{20}{100}$.

We still need to find $p(E \cap F)$.

Since 5, 7 are rel. prime, the numbers divisible by both are also divisible by 35. There are $\lfloor \frac{100}{35} \rfloor = 2$ elements of $\mathbb{Z}^+$ that don't exceed 100 and are divisible by 35. So $p(E \cap F) = \frac{2}{100}$.

So our final answer is

$$p(E \cup F) = \frac{14}{100} + \frac{20}{100} - \frac{2}{100} = \frac{32}{100} = \frac{8}{25}.$$

Exercise C: Monty Hall Problem. Suppose you are game show contestant. You have the chance to win a large prize. You are asked to select one of three doors to open, the prize is behind one of the doors. The procedure is as follows: You select a door. Once you do so, the game show host, who knows what is behind each door, opens one of the other doors and shows you there is nothing inside. Then he asks you wether you would like to switch doors. Should you keep your original choice, switch doors, or does it not matter?

Not releasing answer yet to build suspense.

Exercise D: Example 2 in 7.~~3~~2. Suppose that a die is biased so that 3 appears twice as ofthen as each other number but all other options are equally likely. What is the probability that an odd number appears when we roll the dice?

This is exercise A in the next worksheet.