Worksheet 6.3 and 6.4

Max's Lecture
MATH 55

July 16, 2019

Exercise A (Examples from book). Express each quantity as either a combination or a permutation.

1. The number of ways to select a first prize winner, a second prize winner, and a third prize winner from 100 people at a contest

2. The number of ways to choose which six astronauts to go to mars out of 30 trained people.

3. The number of ways to choose 3 letters from English alphabet. (there are 26 total letters)

4. The number of ways to choose one shirt for me and one for my friend, from my collection of 8 shirts.

5. The number of bit strings of length 8 that contain exactly 3 ones.

Exercise B(6.3.21). How many permutations of the letters ABCDEFGH contain:

1. The string ED?
2. The strings BA and GF
3. The strings ACD and CDE?
4. The strings CBA and BED?

Exercise C (adaoted from 6.3.26. How many ways are there for three penguins and six puffins to stand in line so that:

1. All the puffins stand together

2. All the penguins stand together

3. No two penguins stand next to each other.

Exercise D (Example from book).   1. What is the coeffictint of $x^{12}y^{13}$ in $(x+y)^{25}$

2. What is the coefficient of $x^{12}y^{13}$ in $(2x - 3y)^{25}$

Exercise E(example from book). Prove the identity:

Let n be a positive integer. Then:

$$\sum_{k=0}^{n}(-1)^k \binom{n}{k} = 0$$

For an extra challenge, try to think of a combinatorial proof!