Worksheet 6.1 and 6.2

Max's Lecture
MATH 55

July 15, 2019

Exercise A (Various problems from Rosen and Charles' Worksheets). Use counting techniques to compute the following:

1. How many bit strings are there of length eight?
2. How many bit strings of length eight either start with a 1 or end with 2 zeros?
3. Consider the set of integers 1 through 10. How many subsets contain the number 1 or the number 10?
4. How many subsets contain neither the number 1 nor the number 10?
5. Suppose p and q are prime numbers and $n = pq$. How many numbers not exceeding n are relatively prime to n?

Exercise B (Various sources). Answer each question using pigeon hole principle

1. Show that among any group of 5 integers, there are two that have the same remainder when divided by 4.
2. Show that among any group of 3 integers, there are two whose sum is even.
3. How many distinct numbers must be selected from the set of numbers 1 to 6 to guarantee that at least one pair of these numbers add up to 7?
4. There are 5 points inside an equilateral triangle of side length two centimeters. Show that at least two of the points are within 1 centimeter of each other.
5. Challenge: Show that in a group of n people, there are two with an identical number of friends within the group. (no one is friends with themself). Hint: You will need to consider two separate cases.