

# Math N55– Practice Final Discrete Mathematics

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Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

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This exam contains 9 pages (including this cover page) and 7 questions. Total of points is 50.  
Good luck !

## Distribution of Marks

Question	Points	Score
1	9	
2	9	
3	10	
4	6	
5	6	
6	5	
7	5	
Total:	50	

1. If possible for each of the following give an example. If none exist, explain why.

(a) (3 points) A connected graph with 9 vertices and 7 edges.

(b) (3 points) A bipartite graph with 15 edges.

(c) (3 points) A simple graph that has chromatic number greater than 4.

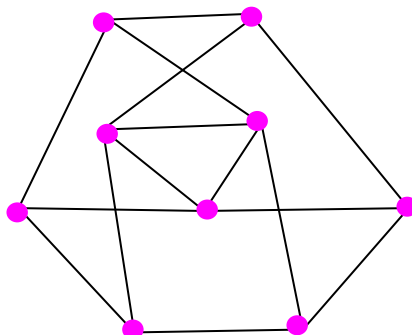
2. Give the cardinality of the following sets. If the cardinality is infinite specify whether countably or uncountably infinite. Justify your answers.

(a) (3 points) The set of reflexive relations on a set of  $n$  elements

(b) (3 points) The set of regions in a planar graph with 4 vertices each of degree two.

(c) (3 points) The set of solutions to the recurrence  $a_n = 2a_{n-1} + 6a_{n-2}$

3. Let  $G$  be the graph shown below:



- (a) (4 points) Show that  $G$  does not have a subgraph homeomorphic to  $K_5$ . Hint: Consider the vertex degrees.
- (b) (3 points) Show that  $G$  is not planar.
- (c) (3 points) What is the chromatic number of  $G$ ? Justify your answer.

4. (6 points) Prove that if  $S$  is a 15 element subset of  $\{1, 2, \dots, 50\}$ , then there are four distinct elements  $a, b, c, d \in S$  such that  $a + b = c + d$

5. (6 points) Find the number of onto functions from the set  $\{1, 2, 3, 4, 5, 6\}$  to the set  $\{a, b, c\}$ .

6. (5 points) Find integers  $y$  and  $z$  such that  $55y + 38z = 1$ .

7. (5 points) Factor the binomial coefficient  $\binom{18}{7}$  as a product of primes.



This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.