Homework Sections 2.3-4.3

Max's lecture
MATH 55

Due July 9, 2019

Note: All problems are taken from Rosen, Discrete Mathematics and its applications, 8th ed. Have fun, and please feel free to ask each other and me for help!

Exercise 2.3.1. Why is f not a function from $\mathbb{R} \to \mathbb{R}$ if:

(a) $f(x) = 1/x$

(b) $f(x) = \sqrt{x}$

(c) $f(x) = \pm\sqrt{x^2+1}$

Exercise 2.3.6abd. Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

(a) the function that assigns to each pair of positive integers the first integer of the pair.

(b) the function that assigns to each positive integer its largest decimal digit

(d) the function that assigns to each positive integer the largest integer not exceeding the square root of the integer.

Exercise 2.3.12. Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one:

(a) $f(n) = n - 1$

(b) $f(n) = n^2 + 1$

(c) $f(n) = n^3$

(d) $f(n) = \lceil n/2 \rceil$

Exercise 2.3.14. Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto:

(a) $f(m, n) = 2m - n$

(b) $f(m, n) = m^2 - n^2$

(c) $f(m, n) = m + n + 1$

(d) $f(m, n) = |m| - |n|$

(e) $f(m, n) = m^2 - 4$

Exercise 2.3.28. Show that the function $f(x) = e^x$ from the set of real numbers to the set of real numbers is not invertible, but if the codomain is restricted to the set of positive real numbers, the resulting function is invertible.

Exercise 2.3.32. Let $f(x) = 2x$ where the domain is the set of real numbers. What is:

(a) $f(\mathbb{Z})$

(b) $f(\mathbb{N})$

(c) $f(\mathbb{R})$

A note on notation: If S is a subset of the domain, then $f(S)$ is the set of $f(s)$ for all $s \in S$.

Exercise 2.3.36. If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.

Exercise 2.5.4. Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

(a) integers not divisible by 3

(b) integers divisble by 5 but not by 7

(c) the real numbers with decimal representations consisting of all 1s

(d) the real numbers with decimal representations of all 1s or 9s.

Exercise 2.5.6. Suppose Hilbert's Grand Hotel is fully occupied, but the hotel closes all the even numbered rooms for maintenance. Show that all the guests can remain in the hotel.

Exercise 2.5.10. Give an example of two uncountable sets A and B such that $A - B$ is:

(a) finite

(b) countably infinite

(c) uncountable

Exercise 2.5.20. Show that if $|A| = |B|$ and $|B| = |C|$, then $|A| = |C|$.

Exercise 2.5.30. Show that the set of real numbers that are solutions of quadratic equations $ax^2 + bx + c = 0$, where a, b, c are integers, is countable. Note: You can use the fact that if $A_1, A_2, \ldots, A_n$ are countable sets, then $A_1 \times \cdots \times A_n$ is a countable set.

Exercise 4.1.10. Prove that if a and b are nonzero integers, a divides b, and $a + b$ is odd, then a is odd.

Exercise 4.1.12. Prove that if a is a positive integer, then 4 does not divide $a^2 + 2$

Exercise 4.1.16c. What time does a 24-hour clock read 168 hours after it reads 19:00?

Exercise 4.1.18ace. Suppose that a and b are integers, $a \equiv 11(\mod 19)$, and $b \equiv 3(\mod 19)$. Find the integer c with $0 \leq c \leq 18$ such that:

(a) $c \equiv 13a(\mod 19)$

(c) $c \equiv a - b(\mod 19)$

(e) $c \equiv 2a^2 + 3b^2(\mod 19)$

Exercise 4.1.44. Show that if n is an integer then $n^2 \equiv 0$ or $1(\mod 4)$.

Exercise 4.2.2c. Convert the decimal expansion of 100632 to binary expansion.

Exercise 4.2.4b. Convert the binary expansion $(10\,1011\,0101)_2$ to a decimal expansion.

Exercise 4.2.7d. convert $(DEFACED)_16$ to a binary expansion

Exercise 4.2.8. Convert $(BADFACED)_16$ from its hexadecimal expansion to its binary expansion.

Exercise 4.2.28. (Note: This problem won't be graded for correctness, since we don't cover the algorithm in class) Use Algorithm 5 to find $123^{1001} \mathbf{\ mod\ } 101$

Exercise 4.2.32. Show that a positive integer is divisible by 11 if and only if the difference of the sum of its decimal digits in even-numbered positions and the sum of its decimal digits in odd-numbered positions is divisible by 11.

Exercise 4.2.38. Find the decimal expansion of the number with n-digit base seven expansion $(11\ldots11)_7$ (with n ones). Hint: Use the formula for the sum of the terms of a geometric progression. We didn't cover this yet in class, but this formula can be found on page 174 in theorem 1.

Exercise 4.3.4f. Find the prime factorization of 899.

Exercise 4.3.6. How many zeros are there at the end of 100!?

Exercise 4.3.10. Show that if $2^m + 1$ is an odd prime, then $m = 2^n$ for some nonnegative integer n. [hint: first show that the polynomial identity $x^m + 1 = (x^k + 1)(x^{k(t-1)} - x^{(k(t-2)} + \cdots - x^k + 1)$ holds, where $m = kt$ and t is odd.]

Exercise 4.3.12. Prove that for every positive integer n, there are n consecutive composite integers. [Hint: consider the n consecutive integers starting with $(n + 1)! + 2$].

Exercise 4.3.18. We call a positive integer perfect if it equals the sum of its positive divisors other than itself.

(a) Show that 6 and 28 are perfect.

(b) Show that $2^{p-1}(2^p - 1)$ is a perfect number when $2^p - 1$ is prime.

Exercise 4.3.22. Show that n is prime if and only if $\phi(n) = n - 1$.

Note: The book states above this problem that $\phi(n)$ is the number of positive integers less than or equal to n that are relatively prime to n.

Exercise 4.3.30. If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?

Exercise 4.3.32ef. Use the Euclidean algorithm to find gcd(1529, 14038) and gcd(11111, 111111).

Exercise 4.3.40ad. Using the method followed in Example 17, express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.

(a) 9,11

(d) 21,55