Homework Sections 1.1-1.5

Max's lecture
MATH 55

Due June 27, 2019

Note: All problems are taken from Rosen, Discrete Mathematics and its applications, 8th ed. Have fun, and please feel free to ask each other and me for help!

Exercise 1.1.2. Which of these are propositions? What are the truth values of those that are poropositions?

(b) What time is it?

(e) The moon is made of green cheese.

(f) $2^n \geq 100$

Exercise 1.1.14. Let p, q, r be the propositions:

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

Express each of the following propositions as an english sentence.

(b) $\neg q \iff r$

(d) $p \vee q \vee r$

(e) $(p \to \neg r) \vee (q \to \neg r)$

Exercise 1.1.16. Let p, q, r be the propositions:

p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in this class.

Write the following propositions using p, q, r and logical connectives (including negations). (So write it using logical symbols)

(a) You get an A in this class, but you do not do every exercise in this book.

(d) You get an A on the final, but you don't do every exercise in this book; nethertheless you get an A in the class.

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

Exercise 1.1.24. Write each of these statements in the form "if p, then q" in English.

(a) It is necessary to wash the boss's car to get promoted.

(b) Winds from the south imply a spring thaw.

(g) Carol gets seasick whenever she is on a boat.

Exercise 1.2.38. 5 friends have access to a chat room. Is it possible to determine who is chatting if the following information is known?

Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Show your reasoning.

Exercise 1.3.8. Use de Morgans laws to find the negation of each of the following statements.

(a) Kwame will take a job in industry or go to graduate school.

(b) Yoshiko knows Java and calculus.

(c) James is young and strong.

(d) Rita will move to oregon or Washington.

Exercise 1.3.10. For each of these compound propositions, use the conditional-disjunction equivalence to find an equivalent compound propositions that does not involve conditionals.

(a) $\neg p \to \neg q$

(b) $(p \vee q) \to \neg p$

(c) $(p \to \neg q) \to (\neg p \to q)$

Exercise 1.3.22. Show that $p \to q$ and $\neg q \to \neg p$ are logically equivalent. (This is showing that an implication means the same thing as its contrapositive!)

Exercise 1.3.30. Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

Exercise 1.4.8. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops", and the domain consists of all animals.

(a) $\forall x(R(x) \to H(x))$

(b) $\forall x(R(x) \wedge H(x))$

(c) $\exists x(R(x) \to H(x))$

(d) $\exists x(R(x) \wedge H(x))$

Exercise 1.4.16. Determine the truth value of each of these statements if the domain for all variables consists of all real numbers.

(a) $\exists x(x^2 = 2)$

(b) $\exists x(x^2 = -1)$

(c) $\forall x(x^2 + 2 > 1)$

(d) $\forall x(x^2 \neq x)$

Exercise 1.5.10. Let $F(x, y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express each of these statements.

(a) Everybody can fool Fred

(d) There is no one who can fool everybody

(i) No one can fool themself.

(j) There is someone who can fool exactly one person besides himself or herself.

Exercise 1.5.22. Use predicates, quantifiers, logical connectives, and mathematical operators to express the statement that there is a positive integer that is not the sum of three squares.

Exercise 1.5.36. Express each of these statments using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Don not simply use the phrase " it is not the case that".)

(e) No student has solved at least one exercise in every section of this book.